Time-varying Wind Speed Identification from Structural Responses by Modified Iterative Method

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Abstract: The aim of this study is to try to identify the time-varying wind speed in longitudinal direction at the referenced level from structural responses. The random fluctuating wind speed process was simulated by spectral representation method and FFT method, together with the power law used to calculate wind load. Based on the vibration equation of system in state space and regularization technique, the wind speed at referenced level can be identified from structural responses. The modified iterative method is proposed to reduce the identified errors induced by the difference of the fluctuating wind speed at different level. A 50 m tall guyed mast is selected as an example, and used to check the validation and effectiveness of the proposed method. Simulated numerical results indicated that the proposed method can be used to identify wind speed values from structural displacement responses or strain responses accurately and effectively. Displacement or strain responses obtained in two orthogonal directions at a reference level only are required for an accurate identification and modified iterative method can improve the identified results obviously.

Keywords: wind speed; identification; dynamic response; regularization; iteration; guyed mast.

1. Introduction

Wind speed measurements generally have been carried out by propeller type anemometers or ultrasonic anemometers in structural system health monitoring (Li et al. 2002 and Pines et al. 2002) or other research works (Harikrishna et al. 2003 and Edmund 2004). In most structure design process, the wind load calculating usually is based on the basic wind pressure induced by the long term records of wind speed and direction data in the area where the structure was built, and their statistical information, considering the surface roughness of the earth, coefficient of structure shape of system for wind load, coefficient of wind pressure variation with height and magnified factor induced by fluctuating wind vibration, etc. Chen and Li (2001) estimate the wind load applied on the shear building system by general statistical average algorithm based on the system responses, including displacements, velocities and accelerations of all degree of freedoms, when structural parameters are unknown. But wind speed identification from structural responses has not been found out in the existing literatures.

However, force identification, especially for moving force identification from bridge responses, has been studied in the last two decades, the researchers have developed many methods (Law et al. 1997, 1999 and 2004) to execute this task. The wind action on a structure is very similar to that with moving vehicles passing on top of a bridge deck. When a proper wind load model is adopted, the responses of the structure can be formulated explicitly as a function of the wind speed, and the wind speed can then be identified from the structural responses in the inverse analysis.

Based on the above discussions, the project of this study is to try to identify the wind speeds in longitudinal direction at the referenced level from structural responses. This may benefit to analyzing the characteristics of wind and wind load. The random fluctuating wind speed process was simulated by spectral representation method and FFT method proposed by Deodatis (1996), together with the power law used to calculate wind load. Based on the vibration equation of system in state space and regularization method, the wind speed at the referenced level can be identified from structural responses. The modified iterative method is proposed to reduce the identified errors induced by the difference of the fluctuating wind speed at different level. A 50 m tall guyed mast was selected as an example, and used to check the validation and effectiveness of the proposed method. The guyed mast was modeled as equivalent space beam for mast and space truss for guys, the
connections between guy and beam elements are assumed to be rigid links, which preserve the guy eccentricities, and static reduced technique used to condense the slave-degrees-freedom. Davenport and Kaimal wind spectral formulations are employed in the fluctuating wind speed simulation, respectively. There are three cases to be discussed, case 1 is a simple one, making use of the simulated fluctuating wind speed from Davenport spectral, assumed the same fluctuating wind speed for all nodes of different levels in direct analysis; case 2 make use of different simulated fluctuating wind speeds from Davenport spectral at each height level in direct analysis; and case 3 accords with the practical situation, the simulated fluctuating wind speed data from Kaimal spectral at different levels are used in the direct problem. All simulated numerical results indicate that the proposed method can be used to identify wind speed values from structural displacement responses or strain responses accurately and effectively. And displacement or strain responses obtained in two orthogonal directions at a reference level only are required for an accurate identification. Modified iterative method can improve the identified results obviously.

2. Calculation of wind load in time domain

The longitudinal wind load is considered in the following analysis neglecting the transverse and vertical wind load components. The wind speed at level \( z \) above ground, \( v(z,t) \), can be written as

\[
v(z,t) = \bar{v}(z,t) + \hat{v}(z,t)
\]

(1)

where \( \bar{v}(z,t) \) and \( \hat{v}(z,t) \) denote the average wind speed and fluctuating wind speed, respectively. The mean wind speed at different level, \( \bar{v}(z) \), may be calculated according to the Power Law (Simiu et al. 1996)

\[
\bar{v}(z) = \bar{v}(z') \left( \frac{z}{z'} \right)^{\alpha}
\]

(2)

in which \( z' \) and \( \bar{v}(z') \) are the reference height and average wind speed at the reference level, respectively. \( z \) and \( \bar{v}(z) \) are arbitrary height and its corresponding average wind speed. The Power Law exponent \( \alpha \) is determined from the terrain roughness, can be found in (Simiu et al. 1996).

The wind load \( F(z,t) \) on the structure at level \( z \) can be written as (Liu, 1991)

\[
F(z,t) = \frac{1}{2} \rho \mu(z) A(z) v^2(z,t)
\]

(3)

where \( \rho \) is the density of air, \( A(z) \) and \( \mu(z) \) are the orthogonal exposed wind area of the structure at level \( z \), and the correlation of shape of the structure at level \( z \), respectively.

Substituting Eqs. (1) and (2) into Eq. (3), yielding

\[
F(z,t) = C_m(z) \bar{v}^2(z') + C_{f1}(z) \bar{v}(z') \hat{v}(z,t) + C_{f2}(z) \hat{v}^2(z,t)
\]

(4)

where \( C_m(z) = \frac{1}{2} \rho \mu(z) A(z) \left( \frac{z}{z'} \right)^{2\alpha} \) is the coefficient of mean wind load, which depends on the reference height and the vertical height of the selected level, \( C_{f1}(z) = \rho \mu(z) A(z) \left( \frac{z}{z'} \right)^{\alpha} \) and \( C_{f2} = \frac{1}{2} \rho \mu(z) A(z) \) are similarly defined coefficients for the fluctuating wind load. Benfratello et al. (1996), after analyzing the stochastic response of a SDOF structure subject to wind action, concluded that neglecting the quadratic pressure term of the fluctuating wind speed could not give accurate results.

The fluctuating wind speed is simulated based on the wind spectrum at different levels, and the Fast-Fourier Transform (Deodatis 1996) is needed to estimate the fluctuating wind speed components acting on the structure. When the mean wind speed \( \bar{v}(z') \) corresponding to a reference level \( z' \) and the time history of fluctuating wind speed at all levels, \( \hat{v}(z,t) \), are obtained, the wind load on the structure can be computed.

In general, the auto-spectral density defined as following two cases, one is proposed by Davenport (1961), the values of auto-spectral density don’t vary with height above the ground.
\[ S(\omega) = \frac{1}{2} \omega u_*^2 \left( \frac{1200\omega}{2\pi\nu(10)} \right)^2 \left( 1 + \left( \frac{1200\omega}{2\pi\nu(10)} \right)^2 \right)^{-\frac{4}{3}} \]  

(5)

where \( \nu(10) \) is mean speed at 10 m level, the other parameters are as same as previous section. \( u_* \) is wind friction speed and calculated as

\[ u_* = \frac{k\nu(z')}{\ln(z'/z_0)} \]

(6)

where \( k \) is a constant, can be approximately taken as 0.4, \( z_0 \) is the roughness length and varies vary with the types of terrains (Simiu et al. 1996).

Another one is proposed by Kaimal etc. (1972), the values of auto-spectral density vary with height above the ground.

\[ S(z, \omega) = \frac{1}{2} \frac{105u_*^2}{2\pi \nu(z)} \frac{z}{\nu(z)} \left( 1 + 33 \frac{z\omega}{2\pi\nu(z)} \right)^2 \]

(7)

3. Wind speed identification

3.1. State equation

The equation of motion can be set up by finite element method

\[ M\ddot{X} + C\dot{X} + KX = P \]

(8)

where \( M_{na} \), \( C_{na} \) and \( K_{na} \) are the mass, damping and stiffness matrices. \( X, \dot{X} \) and \( \ddot{X} \) are \((n \times 1)\) nodal displacements, velocities and accelerations vectors of the structure. \( n \) is the number of degree-of-freedom of the structure. Assuming the wind speed (including fluctuating component) along the vertical direction of the structure having power law (this assumption will be corrected in modified iterative method), the system force vector \( P_{na}^i \) is approximately defined as following in inverse problem.

\[ P = [C_1 \ C_2 \ \ldots \ C_j \ \ldots \ C_n]^T \nu'(z', t) = C\nu \]

(9)

in which \( C = [C_1 \ C_2 \ \ldots \ C_j \ \ldots \ C_n]^T \), \( \nu = \nu^2(z', t) \), \( C_j = \frac{1}{2} \rho \mu_j A_j \left( \frac{z_j}{z} \right)^{2a_j} \), \( j = 1, 2, \ldots, n \), \( A_j \) and \( z_j \) are wind areas and height of the \( j \)-th node. If wind attack angle \( \beta \) (angle between the positive direction of \( x \) axle and wind direction) is not zero, the force vector may be written as

\[ P = \begin{bmatrix} C_{x1} & 0 & \ldots & C_{xj} & 0 & \ldots & C_{xn} & 0 \\ 0 & C_{y1} & \ldots & 0 & C_{yj} & \ldots & 0 & C_{yn} \end{bmatrix} \begin{bmatrix} \nu_{x1}^2(z', t) \\ \nu_{xj}^2(z', t) \\ \nu_{xn}^2(z', t) \\ \nu_{y1}^2(z', t) \\ \nu_{yj}^2(z', t) \\ \nu_{yn}^2(z', t) \end{bmatrix} = C\nu \]

\( C_{xyj} \) and \( C_{yxj} \) can be defined similarly,

\[ \nu_j(z', t) = v(z', t) \cos \beta \] \( \nu_j(z', t) = v(z', t) \sin \beta \)

\( v(z', t) \) is wind speed that corresponds the referred level \( z' \).

Eq. (8) can be written in state space form as

\[ \dot{Z} = KZ + \hat{B}P \]

(10)

where

\[ Z = \begin{bmatrix} X \\
\dot{X} \end{bmatrix}_{2na} \]

\[ K = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}_{2na \times 2na} \]

\[ \hat{B} = \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix}_{2na} \]

If the response of the structure is represented by \( N \) output quantities in the output vector \( y(t) \) from sensors such as accelerometers, velocity transducers, displacement transducers or strain gauges, etc., an output equation can be expressed as
\[ y = R_a \ddot{X} + R_v \dot{X} + R_d X \]  

(11)

where \( R_a, R_v \), and \( R_d \) are output influence matrices for acceleration, velocity, displacement measurements, respectively. Extracting \( \dot{X} \) from Eq. (8) and substituting into Eq. (11) to yield

\[ y = RZ + D\bar{C}\bar{v} \]  

(12)

where \( R = [R_d - R_v M^{-1} \text{K} \quad R_v - R_a M^{-1} \text{C}] \), \( D = R_a M^{-1} \).

Eqs. (10) and (12) are converted into discrete equations using the exponential matrix, and the final discrete model is

\[ Z(j + 1) = AZ(j) + B\bar{C}\bar{v}(j) \]  

(13)

\[ y(j) = RZ(j) + D\bar{C}\bar{v}(j) \quad (j = 1, 2, \ldots, N) \]

where \( N \) is the total number of sampling points. \( \tau \) is the time step between the variable state variables \( Z(j+1) \) and \( Z(j) \), and \( A = \exp(\bar{K}\tau) \), \( B = \bar{K}(A - I)\tilde{B} \).

Solving for the output \( y(j) \) with zero initial conditions from Eq. (13) in terms of the previous inputs

\[ P(i) = \bar{C}\bar{v}(i), (i = 1, 2, \ldots, j) \]

yields

\[ y(j) = \sum_{i=0}^{j} H_i \bar{C}\bar{v}(j - i) \]  

(14)

where \( H_0 = D \) and \( H_i = RA^{(i-1)}B \).

The constant matrices in the series are known as system Markov parameters. The Markov parameters are commonly used as the basis for identifying mathematical models in linear dynamic systems. The Markov parameters represent the response of the discrete system to unit impulse, and they must be unique for the system (Juang 1994).

### 3.2 Wind speed identification with regularization method

Rewrite Eq. (14) to give the matrix convolution equation as

\[ \Pi V = Y \]  

(15)

in which

\[ \Pi = \begin{bmatrix} H_0 \bar{C} & 0 & \cdots & 0 \\ H_1 \bar{C} & H_0 \bar{C} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ H_{N-1} \bar{C} & H_{N-2} \bar{C} & \cdots & H_0 \bar{C} \end{bmatrix}, \quad V = \left\{ \bar{v}(0)^T, \bar{v}(1)^T, \ldots, \bar{v}(N-1)^T \right\}^T, \quad Y = \left\{ y(0)^T, y(1)^T, \ldots, y(N-1)^T \right\}^T \]

This is an ill-posed problem due to the lack of continuous dependence of the solution on the data. A straightforward least-squares solution may produce an unbounded result. Regularization technique would provide an analysis on the ill-posed problem. One approach to regularization proposed by Tikhonov et al. (1977) is to replace Eq. (15) with the associated equation

\[ (\Pi^T \Pi + \lambda I) V = \Pi^T Y \]  

(16)

For \( \lambda > 0 \), the matrix operator \( (\Pi^T \Pi + \lambda I) \) is unique, and therefore, its inverse is continuous. Solving Eq. (16) is equivalent to solve the following unique problem

\[ \min J(V, \lambda) = \|\Pi V - Y\|^2 + \lambda \|V\|^2 \]  

(17)

It is clear from the second term that the non-negative regularization parameter has the effect of forcing a bounded solution. The optimal regularization parameter is determined by minimizing the identified error calculated by Eq. (19) in numerical simulation. But in practical case, the real value is not known, L-curve method proposed by Hansen (1992) can be used to solve the optimal regularization parameter. If the combination of the measured displacements, velocities, strains and accelerations is used to identify the wind speed, each response component in the vector \( y(t) \) in Eq. (11) should be scaled by their respective norms to have dimensionless units.
4. Numerical simulation and results

Here we select a 50m tall-guyed mast as an example to check the validation of the proposed wind speed identification method. White noise is added to the calculated responses of the system to simulate the polluted measurements.

\[ y = y_{\text{calculated}} + E_p \times N_{\text{oise}} \times \sigma(y_{\text{calculated}}) \]  

(18)

where \( y \) is simulated measured responses, \( y_{\text{calculated}} \) is calculated responses, \( E_p \) is noise level, \( N_{\text{oise}} \) is standard normal distribution vector with zero mean value and unit standard deviation and \( \sigma(y_{\text{calculated}}) \) is standard deviation of the calculated responses.

The errors in the identified wind speeds can be calculated using following formulation

\[ \text{Error} = \left\| \frac{v_{\text{identify}} - v_{\text{true}}}{v_{\text{true}}} \right\| \times 100\% \]  

(19)

where \( \| \cdot \| \) is the normal of the wind speed vector. \( v_{\text{identify}} \) and \( v_{\text{true}} \) are identified and true wind speed time history, respectively.

4.1 Computing model of the guyed mast

4.1.1 Introduction of the 50m-tall guyed mast system

The 50m tall guyed mast has an equilateral triangular cross-section, and its side length is 1.2 m. The diagonals and horizontal members are 60mm steel tubes of 5mm thickness. The mast is divided into 43 vertical segments with the first 40 segments 1.16m high each and the others 1.2m high. It is tied to the ground by cables at two levels of 20.88 and 46.40m above ground level with three cables at each level as shown in Fig. 1. The cables are of 17.5mm diameter each consists of 19 numbers of 3.55mm diameter galvanized steel wires, and the initial tension in each cable is 23520 N. The Young’s modulus of material is 2.05×10¹¹ N/m² and 1.2×10¹¹ N/m² for the mast and cables, respectively. The mass density and Poisson ratio for the mast are 7850 kg/m³ and 0.25 respectively while those for the cables are 7820 kg/m³ and 0.20 respectively. Three reinforced concrete blocks are provided at a radial distance of 30m from the center of the mast serving as reaction anchors, and they are located along lines radiated from the center of cross-section passing through one of the vertical legs. Cables aligned in one vertical plane are anchored to one of the reaction block. The mast is supported on a reinforced concrete pedestal founded on firm soil.

The coordinate system used in the following analysis is referred to Figure 1. The Z- axle passes through the center of gravity of cross-section of the guyed mast, and the origin of coordinates is set at the ground level.

4.1.2 Equivalent model of the guyed mast

![Fig. 1 Schematic view of the instrumented guyed mast](image-url)
The guyed mast is modeled as an assembly of three-dimensional beam-column elements that approximately describe groups of similar panels of the mast. The initial axial force and shear effect are considered in the elements.

The equivalent beam-column properties of the mast can be derived by using the unit load method to determine the displacements of its centurial axis under separate axial ($V_z$), shear ($V_x$ and $V_y$), bending ($M_x$ and $M_y$) and torsional ($M_z$) loads and equate them to those obtained from a beam-column model analysis under the same forces. These can be derived using an element end flexibility matrix approach based on the principal of virtual work. The guys are modeled as three-dimensional truss elements. Element $C_1M$ in Fig. 2 is modeled as a truss element with rigid arm (Kahla 1995). Node $S_1$ is an inner or slave node and node $M$ is the global or master node. $S_1M$ is the rigid arm is used to represent the connection between cable and beam element.

4.1.3 Sag effect of guys and beam-column effect of mast

The equivalent elastic modulus $E_{eq}$ is adopted to include the sag effect of guy. It is calculated from the formulation proposed by Ernst (1965) as

$$E_{eq} = \frac{E}{1 + \frac{w^2L^2AE}{12T^3}}$$

(20)

where $E$ is the original elasticity modulus of guys; $L$ is the cable length in the horizontal plane; $w$ is the weight per unit length; $A$ is the cross-sectional area of the cable and $T$ is the initial tension force of the cable. The equivalent elastic modulus calculated from the above formulation is the tangent modulus.

Geometry stiffness matrix (Raman et al. 1987) is included in the finite element model to consider the beam-column effect from the axle force and moment acting on the element induced by the initial tension force in the guys.

4.1.4 Reduced model of the system

Guyan static reduction (Guyan 1965) is adopted to condense the slave DOFs to the master DOFs of the guyed mast. All the measured DOFs are designated as the master DOFs and are denoted by $X_m(t)$. The remaining structural DOFs are called the slave DOFs, and are denoted by $X_s(t)$. The equation of motion of the reduced structural model is written as

$$M_r\ddot{X}_m + C_r\dot{X}_m + K_rX_m = P_m$$

(21)

where $M_r$, $C_r$, and $K_r$ are the condensed mass, stiffness and damping matrices corresponding to the master DOFs.

The system is discreted as 6 special truss elements and 10 special beam elements and nodal coordinates can be seen in Figure 1. The equivalent properties of the special beam elements calculated according to reference (Kahla 1995) for a mast with equilateral triangular cross-section. In the following studies, $X$ and $Y$ directions of all mast nodes are taken as master DOFs and the others are slaves. With the assumption of Rayleigh damping, the damping ratio is taken as 2%, and the system vibration equation can be set up.

4.2 Simulated fluctuating wind speed

In the fluctuating wind simulation, the truncating frequency, namely highest frequency, is taken as 2.5 Hz and the number used to divide the truncated frequency equally is 2048, time step is 0.2s. The roughness length $z_0$ is taken as 0.02, and the exponential decay coefficient in horizontal direction $C_z$ is taken as 8. We consider the wind profile obeys power law and the power law exponent is 0.16. For convenience considering, the
reference height is assumed as 50m that equals the height of node No. 10, mean wind speed is 30m/s corresponding referenced level and the angle between the wind direction and the positive direction of X axle is assumed as 30 degrees. Density of air is selected as 1/815 kg/m³ in wind load calculating.

We make use of Davenport and Kaimal spectral calculated by Eqs. (5) and (7) to simulate fluctuating wind speed, respectively. Figs. 3 and 4 show the simulated fluctuating wind speed time history at node 1, 4, 7 and 10 for 10 minutes time.

Fig. 3 Simulated fluctuating wind speeds from Davenport spectral

Fig. 4 Simulated fluctuating wind speed from Kaimal spectral
4.3 Identified results and discussion

Wind area calculating is based on reference (Kahla 1995), and wind load as applied external force is assumed to concentrate in node. The responses of the system are calculated using a time increment of 0.02 second, and the wind data is interpolated to have the same time increment, namely, the sampling frequency is 50 Hz, in the direct and inverse problem.

We select three cases to be studied, case 1 is a simple case, making use of the simulated fluctuating wind speed from Davenport spectral, assumed the same fluctuating wind speed for all nodes of different levels in direct analysis; case 2 make use of different simulated fluctuating wind speeds from Davenport spectral at each height level in direct analysis; and case 3 accords with the practical situation, the simulated fluctuating wind speed data from Kaimal spectral at different levels are used in the direct problem. The initial displacements and velocities of the system all are taken as zeros and the mean wind speed profile for three cases all are assumed to obey the power law.

As a matter of fact, we can’t obtain the responses of the system in field test from zeros initial condition because of the wind is continuous. But the velocity and acceleration responses of structure can’t reveal the effect induced by mean wind speed from non-zeros initial condition. So below we only analyze the identification from displacement responses.

4.3.1 Validation of the proposed method

Case 1: Firstly, in order to check the effect of locations and numbers of measurements stations on the identifications, we make use of noise-free responses to identify the wind speed in x and y directions. Two displacements in X and Y directions of each node used to identify the wind speed at referenced level. Table 1 gives the identified results form displacements of 16 sets and Fig. 5 shows the results identified from the responses of node No. 8 and 10 (sets 8 and 10).

![The wind speed component in x direction](image)

![The wind speed component in y direction](image)

**Fig. 5 The identified results from displacements of node No. 8 and 10 for Case 1**

(— True wind speed, …— From node No. 8, ……From node No. 10)

From Table 1 and Fig. 5, we can obtain following observations:

There are not significant differences of the identified results from different displacement response sets except from node No. 1, multi-node responses can’t give better results. The results identified from the responses of node No. 8 are the best. Displacements responses in X and Y two directions of one node is enough to identify two components of wind in X and Y directions accurately.

The identified errors mainly come from the first few seconds, this may be due to the responses induced by wind is not stable at beginning (zeros initial condition) because of damping existing in the system.
### Table 1 – The identified results from displacement responses in X and Y directions for case 1

<table>
<thead>
<tr>
<th>No. of responses set</th>
<th>No. of node</th>
<th>Height (m)</th>
<th>Error of (v_x) (%)</th>
<th>Error of (v_y) (%)</th>
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<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>5.80</td>
<td>2.85</td>
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<td>2.88</td>
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</tr>
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<tr>
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<td>2.51</td>
</tr>
<tr>
<td>16</td>
<td>1, 2, 3, 4, 5, 6, 7, 8, 9, 10</td>
<td>-</td>
<td>2.68</td>
<td>2.72</td>
</tr>
</tbody>
</table>

Note: (1) * refers guys tied nodes.
(2) - refers no corresponding values.
(3) \(v_x\) and \(v_y\) are components of wind speed in \(X\) and \(Y\) directions, respectively.

Secondly, displacement responses of node No. 10 with different noise level are used to identify the wind speed, the identified results are listed in Table 2, and Fig. 6 shows the identified results from responses of node No. 10 with 1% and 5% noise level.

### Table 2 – The identified results from displacements of node 10 with different noise level

<table>
<thead>
<tr>
<th>Noise level (%)</th>
<th>Error of (v_x) (%)</th>
<th>Error of (v_y) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.55</td>
<td>2.58</td>
</tr>
<tr>
<td>1</td>
<td>5.09</td>
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</tbody>
</table>

Fig. 6 The identified results from noised displacements of node No. 10 for Case 1

(— True wind speed,--- With 1% noise, ……With 10% noise)
From Table 2 and Fig. 6, we can know that identified error increases with noise level, but it is not very sensitive to noise, and the noise induces higher components into the responses, further induces higher components into the identified wind speed in inverse problem.

4.3.2 Modified iterations for an improved identification

Case 2: Making use of 15 displacement responses sets to identify the wind speed components in X and Y directions, Table 3 lists the relative percent errors of the identified results, and Fig. 7 shows the results from displacements of node No. 10 and 6.

<table>
<thead>
<tr>
<th>No. of responses set</th>
<th>No. of node</th>
<th>Error of $v_x$ (%)</th>
<th>Error of $v_y$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>33.17</td>
<td>27.39</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>27.66</td>
<td>23.01</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>22.95</td>
<td>19.42</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>18.13</td>
<td>15.71</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>15.38</td>
<td>13.42</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>13.32</td>
<td>11.70</td>
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<tr>
<td>7</td>
<td>7</td>
<td>11.57</td>
<td>10.38</td>
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<td>8</td>
<td>9.57</td>
<td>9.10</td>
</tr>
<tr>
<td>9</td>
<td>9</td>
<td>7.57</td>
<td>8.05</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>6.62</td>
<td>7.67</td>
</tr>
<tr>
<td>11</td>
<td>9, 10</td>
<td>7.04</td>
<td>7.82</td>
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<tr>
<td>12</td>
<td>8, 9, 10</td>
<td>7.71</td>
<td>8.13</td>
</tr>
<tr>
<td>13</td>
<td>7, 8, 9, 10</td>
<td>8.37</td>
<td>8.45</td>
</tr>
<tr>
<td>14</td>
<td>6, 7, 8, 9, 10</td>
<td>9.35</td>
<td>8.98</td>
</tr>
<tr>
<td>15</td>
<td>1, 2, 3, 4, 5, 6, 7, 8, 9, 10</td>
<td>9.71</td>
<td>9.20</td>
</tr>
</tbody>
</table>

Note: $v_x$ and $v_y$ are components of wind speed in X and Y directions, respectively.

From Table 3 and Fig. 7, following observations can be educed:

It is interesting that the identified wind speed data almost is less than the true of all time history. The identified accuracy becomes higher when the measured station is closer to the referenced level, but when the number of measured station increases, the identified results don’t become well, on the contrary, become worse. This may be due to the difference of fluctuating wind speed time history between each other.

Eq. (4) is used to calculate the responses of the system in direct problem based on wind speeds of all nodes, and Eq. (9) assumes a Power Law distribution of the wind speed based on the wind speed at the reference level in inverse analysis. In fact, there is difference in the fluctuating wind speeds at different levels, even through
they have the same power spectral density. This difference may induce large identified error. In order to solve this problem, the modified iterative approach is therefore modified as follows:

Step 1: From the measured responses to identify the wind speed at referenced level (including mean wind speed and fluctuating wind speed, namely, \( \mathbf{v}(z', t) = \mathbf{\bar{v}}(z') + \mathbf{\hat{v}}(z', t) \)) by Eq. (16).

Step 2: Calculating the auto-spectral density \( S(\omega, z') \) of the identified mean wind speed \( \mathbf{v}(z', t) \) corresponding the referenced height level \( z' \) by Eq. (5).

Step 3: Simulating the fluctuating wind speeds of all nodes at different height.

Step 4: Computing the wind load by Eq. (4) from the identified mean wind speed and the simulated fluctuating wind speed data, and obtaining the responses corresponding the measured freedoms by solving the motion equation of the system.

Step 5: Re-identifying the wind speed increment \( \Delta \mathbf{v}(z', t) \) based on the difference between the measured and the calculated responses last step.

Step 6: Based on the new identified wind speed \( \mathbf{v}_j(z', t) = \mathbf{v}_{j-1}(z', t) + \Delta \mathbf{v}_j(z', t) \), including two parts \( \mathbf{v}_j(z') = \mathbf{v}_{j-1}(z') + \Delta \mathbf{v}_j(z') \), \( \mathbf{\hat{v}}_j(z', t) = \mathbf{\hat{v}}_{j-1}(z', t) + \Delta \mathbf{\hat{v}}_j(z', t) \), repeat step 2 to 5 till the two following conditions are satisfied.

\[
\left| \frac{\Delta \mathbf{v}_j(z')}{\mathbf{v}_j(z')} \right| \leq T_{\text{tolerance}1} \quad \text{and} \quad (1 - c_e) \leq T_{\text{tolerance}2}
\]

where \( c_e \) is correlation coefficients between the identified fluctuating wind speed of adjacent steps, \( \mathbf{\hat{v}}_j(z', t) \) and \( \mathbf{\hat{v}}_{j-1}(z', t) \). The convergence criterion \( T_{\text{tolerance}1} \) and \( T_{\text{tolerance}2} \) all are selected as \( 10^{-6} \) here.

The latest \( \mathbf{v}_j(z', t) \) can be taken as wind speed of referenced height level, and for the others level \( z \) above ground, the wind speed can be calculated as

\[
\mathbf{v}(z, t) = \mathbf{\bar{v}}(z') \left( \frac{z}{z'} \right)^\alpha + \mathbf{\hat{v}}(z, t)
\]

\[ (22) \]
where $\tilde{v}_j(z')$ is identified mean wind speed at the latest iteration, and $\hat{v}_j(z,t)$ is the simulated fluctuating wind speed at the latest step.

The identified results from displacements of node No. 10 before and after modified are plotted in Fig. 8. It can be seen clearly that the modification iterations can observably improve the identified results. The modified results match the true time history very well except one or two peaks.

**4.3.3 Further study on the proposed method**

**Case 3** In fact, the wind spectral values vary with the height, because there are differences between the wind speeds of different heights. So in this case, make use of the simulated fluctuating wind speed time history from Kaimal wind spectral.

Fig. 9 shows the identified results from displacements of node No. 6 and 10. The identified relative percent errors from 15 displacement response sets are tabulated in Table 4.

**Table 4 – The identified results from displacement responses for case 3**

<table>
<thead>
<tr>
<th>No. of responses set</th>
<th>No. of node</th>
<th>Error of $v_x$ (%)</th>
<th>Error of $v_y$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>32.08</td>
<td>26.71</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>26.91</td>
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</tr>
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<td>9</td>
<td>7.47</td>
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<tr>
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<td>10</td>
<td>6.40</td>
<td>7.60</td>
</tr>
<tr>
<td>11</td>
<td>9,10</td>
<td>6.88</td>
<td>7.78</td>
</tr>
<tr>
<td>12</td>
<td>8,9,10</td>
<td>7.61</td>
<td>8.11</td>
</tr>
<tr>
<td>13</td>
<td>7,8,9,10</td>
<td>8.32</td>
<td>8.45</td>
</tr>
<tr>
<td>14</td>
<td>6,7,8,9,10</td>
<td>8.88</td>
<td>8.73</td>
</tr>
<tr>
<td>15</td>
<td>1,2,3,4,5,6,7,8,9,10</td>
<td>9.69</td>
<td>9.20</td>
</tr>
</tbody>
</table>

Note: $v_x$ and $v_y$ are components of wind speed in $X$ and $Y$ directions, respectively.

**Fig. 9 The identified results from noise-free displacements for Case 3**

(— True wind speed, ... From node No. 10, ... From node No. 6)
Compare the identified results in Table 4 with those in Table 3, Figs. 10 and 8, we can obtain similar conclusion as case 2, and the identified results in Table 4 almost equal with those in Table 3 corresponding the same response set.

The modification iteration is used to improve the identified results in this case, only the step 2 of above mentioned iterations need to be changed, namely, calculating the new auto-spectral at each node from the identified mean wind speed at the referenced level and power law, by Eqs. (2) and (7).

Fig. 10 gives the identified results from displacements of node No. 10 before and after modified, the improved effect can be found from Fig. 10 obviously.

5. Conclusions

The following conclusions and suggestions for further research may be deduced from above numerical simulations.

1) It is practicable and effective to use structural responses data to identify the wind speed at the referenced level by the proposed method.

2) Two components of wind speed in X and Y directions can be identified accurately from displacements in X and Y directions of the node that corresponds the referenced level for all three cases, and the identified accuracy doesn’t increase with number of measured stations.

3) When the fluctuating wind speed is considered to vary with the height, namely for practical case, modification iterations can help us to obtain well identification results.

4) Only displacement or strain responses of the system with non-zeros initial condition can be used to identify wind speed, because they include the static components induced by mean wind speed, but velocity and acceleration responses do not. Strain responses has not been mentioned previously, but authors of this paper have experience from the research work of moving force identification from bridge responses that strain responses can give same accuracy as displacement responses. And the strain response measurements are easier to obtain than displacement in many cases.

5) The works of time-varying wind speed identification in this paper just is a preliminary study, it may benefit to analyzing the characteristics of wind and wind load. There are many works need to do further, future research should be conducted to field test study, new computation method, and so on.

6. References


